Lösungen zu Klausur Regelungstechnik I (SS13)

Aufgabe 1

a) $u_a,\ v$

b) $\boldsymbol{x} = [i_a\ \omega]^T, \qquad \dot{\boldsymbol{x}} = \begin{bmatrix} -\frac{R_a}{L_a} i_a - \frac{k_1 \Phi_e}{L_a}\omega + \frac{1}{L_a} u_a \\ \frac{k_2 \Phi_e}{\Theta} i_a - \frac{mgr}{\Theta} - \frac{\mu_R}{\Theta}\omega^3 \end{bmatrix}, \qquad y = v = r\omega$

c) $\omega_R = \sqrt[3]{\frac{k_2 \Phi_e}{\mu_R} i_R - \frac{mgr}{\mu_R}}, \qquad u_R = i_R R_a + k_1 \omega_R \Phi_e = i_R R_a + k_1 \sqrt[3]{\frac{k_2 \Phi_e}{\mu_R} i_R - \frac{mgr}{\mu_R}}\,\Phi_e$

d) $\Delta\dot{\boldsymbol{x}} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{k_1 \Phi_e}{L_a} \\ \frac{k_2 \Phi_e}{\Theta} & -\frac{\mu_R}{\Theta} 3\omega_R^2 \end{bmatrix} \Delta\boldsymbol{x} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} \Delta u, \qquad \Delta y = \begin{bmatrix} 0 & r \end{bmatrix} \Delta\boldsymbol{x}$

Aufgabe 2

a) siehe Abbildung 1

b) siehe Abbildung 1 ($\omega_c = 0.25\,\text{rad/s}$)

c) $\arg(\hat{g}(\mathrm{i}\omega)) = 2\arctan(\frac{\omega}{10}) - \frac{\pi}{2} - 2\arctan(\omega) - \arctan(\frac{\omega}{200})$

d) $\hat{g}_r(s) = \frac{V_I(1+T_I s)}{s}$ mit $T_I = 4(2+\sqrt{3})\,\text{s}$ und $V_I = \frac{25}{76\sqrt{2+\sqrt{3}}}$

e) A → IV, B → II, C → I, D → III

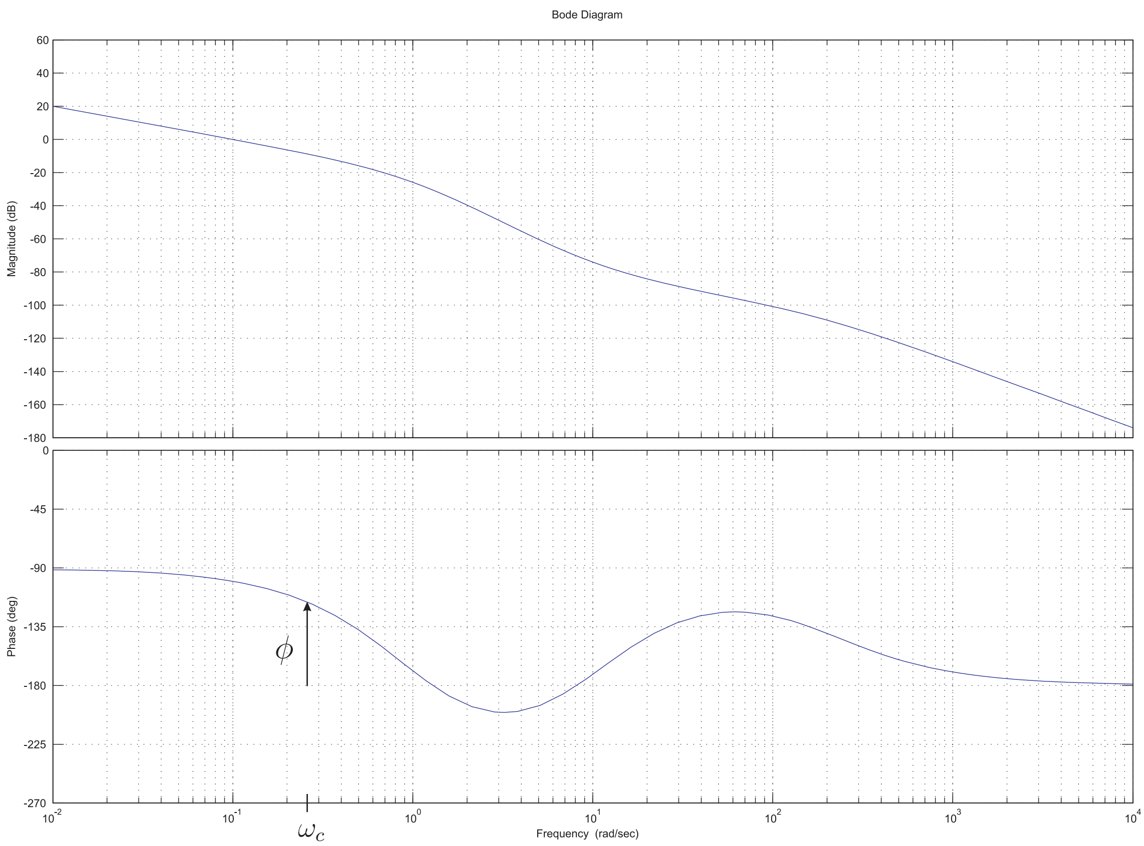

Abb. 1: Bodediagramm zu Aufgabe 2.

Aufgabe 3

a) $A_c = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}, \quad \boldsymbol{b}_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{c}_c^T = \begin{bmatrix} -1 & 1 \end{bmatrix}$

b) $A = VA_cV^{-1}, \quad \boldsymbol{b} = V\boldsymbol{b}_c, \quad \boldsymbol{c}^T = \boldsymbol{c}_c^T V^{-1}, \quad$ wobei $\quad V = V^{-1} = \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}$

c) $\boldsymbol{k}^T = [\,5 \quad 8\,]$, Eigenwerte von $(A - \boldsymbol{b}\boldsymbol{k}^T)$ $\lambda_1 = -3$ und $\lambda_2 = -5$

d) $\det(O(\boldsymbol{c}^T, A)) = 3 \neq 0$

e) $\boldsymbol{l} = \frac{1}{3}\begin{bmatrix} -38 \\ 182 \end{bmatrix}$

f) $p_{gesamt}(\lambda) = (\lambda + 3)(\lambda + 4)(\lambda + 5)(\lambda + 6)$

Aufgabe 4

a) i) $\hat{g}(s) = \frac{-2s - s^2 + 2s^3 + s^4}{1 + 4s + 7s^2 + 4s^3 + s^4}$

ii) $\begin{array}{c|ccc} s^4 & 1 & 7 & 1 \\ s^3 & 4 & 4 & 0 \\ \hline s^2 & 6 & 1 & 0 \\ s^1 & \frac{20}{6} & 0 & 0 \\ s^0 & 1 & 0 & 0 \end{array}$ $\Rightarrow$ Hurwitz-Polynom / System stabil

iii) $y(0) = 1, \qquad y(t \to \infty) = 0$

b) $-4\pi \neq [3 - 3 + 0]\pi \quad \Rightarrow$ geschlossener Regelkreis instabil

c) i) $\hat{y}_1 = \frac{\hat{g}_d\hat{d} + \hat{g}_1\hat{u}_2 - \hat{g}_1\hat{g}_{r1}\hat{g}_n\hat{n}}{(1 + \hat{g}_1\hat{g}_{r1})}$

ii) $\hat{t}_{u_2,y_1} = \frac{\hat{g}_1}{1 + \hat{g}_1\hat{g}_{r_1}}, \qquad \hat{t}_{n,y_1} = \frac{-\hat{g}_1\hat{g}_{r1}\hat{g}_n}{1 + \hat{g}_1\hat{g}_{r_1}}$

iii) siehe Abbildung 2

iv) $\hat{t}_{r,y_2} = \frac{\hat{g}_2\hat{t}_{u_2,y}\hat{g}_v}{1 + \hat{g}_2\hat{t}_{u_2,y}\hat{g}_{r2}}$

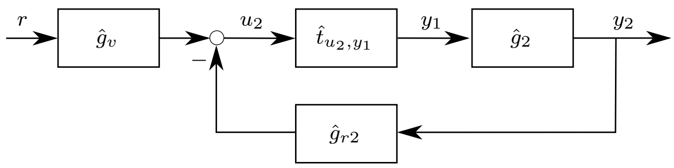

Abb. 2: Blockschaltbild zu Aufgabe 4.